Chapter 5 Gases (<u>Chapter 9</u> in OpenStax)

- Unlike solids and liquids, gases can compress and expand easily.
- Their volume changes are controlled by temperature (T), pressure (P), and moles (n).

## Pressure

- Pressure equals force per unit area:  $P = \frac{F}{A}$
- Force is exerted by gases due to limitations on volume (e.g., due to gravity).
- For P due to gravity, suppose an object has  $m = 1.0 \times 10^{-3}$  kg and  $A = 5.00 \times 10^{-4}$  m<sup>2</sup>  $F = mg = (1.0 \times 10^{-3} \text{ kg})(9.81 \text{ m/s}^2) = 9.81 \times 10^{-3} \text{ kg} \cdot \text{m/s}^2 = 9.81 \times 10^{-3}$  Newtons (N)  $P = \frac{F}{A} = \frac{0.00981 \text{ N}}{5.00 \times 10^{-4} \text{ m}^2} = 19.6 \frac{\text{N}}{\text{m}^2} = 19.6 \frac{\text{kg}}{\text{m} \cdot \text{s}^2} = 19.6$  Pascals (Pa)

 $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 101.3 \text{ kPa} = 1.013 \text{ bar} = 760 \text{ mmHg} = 14.696 \text{ psi}$ 

Ex 5.01 Using Conversion Factors for Pressure

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$$(6.21 \text{ atm})\left(\frac{760 \text{ mmHg}}{1 \text{ atm}}\right) = 4.72 \times 10^3 \text{ mmHg}$$

- 
$$(4.72 \times 10^3 \text{ mmHg}) \left(\frac{101.3 \text{ kPa}}{760 \text{ mmHg}}\right) = 629 \text{ kPa}$$

Barometers/Manometers

- <u>Pressure of air</u> and other gases is measured by increase in height of a liquid in a column.
- The <u>barometer</u> equation P = gdh can be derived from  $P = \frac{F}{A}$ , F = mg,  $A = \frac{V}{h}$  and  $V = \frac{m}{d}$ .

- 
$$P = \frac{F}{A} = \frac{mg}{V/h} = \frac{mgh}{V} = \frac{mgh}{(m/d)} = \frac{gh}{(1/d)} = gdh$$

- For water,  $d = 1000 \text{ kg/m}^3$
- With height (h) in meters, the result for pressure (P) is in units of Pa (or  $N/m^2$ ).
- Use 760 mmHg =  $1.01 \times 10^5$  Pa to convert the pressure into mmHg.

Boyles' Law (Robert Boyle, 1661)

- Volume is inversely proportional to P at constant T. That is:  $V \alpha 1/P$ .
- PV = constant (at constant T and moles).

- 
$$P_i V_i = P_f V_f$$
 Alternate equations:  $P_f = P_i \left(\frac{V_i}{V_f}\right)$  and  $V_f = V_i \left(\frac{P_i}{P_f}\right)$ 

Ex 5.02 Using Boyle's Law to Determine Volume when Pressure Changes

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$$V_f = V_i \left(\frac{P_i}{P_f}\right) = (32.7 \text{ dm}^3) \left(\frac{85.0 \text{ kPa}}{101.3 \text{ kPa}}\right) = 27.4 \text{ dm}^3 = 27.4 \text{ L}$$

Charles' Law (Jacques Charles, along with J. Dalton and J.L. Gay-Lussac, 1787)

- <u>V  $\alpha$  T</u> (at constant P and moles) if T is in Kelvins (K =  $^{\circ}C + 273$ ), so that V/T = constant.
- $\frac{V_f}{T_f} = \frac{V_i}{T_i}$  Alternate equations:  $V_f = V_i \left(\frac{T_f}{T_i}\right)$  and  $T_f = T_i \left(\frac{V_f}{V_i}\right)$

Ex 5.03 Using Charles' Law to Determine Volume when Temperature Changes

- $T_i = 10 \ ^{o}C + 273 = 283 \ K \ T_f = 36 \ ^{o}C + 273 = 309 \ K$
- $V_f = V_i \left(\frac{T_f}{T_i}\right) = (27.4 \text{ L}) \left(\frac{309 \text{ K}}{283 \text{ K}}\right) = 29.9 \text{ L}$

Combined Gas Law

- <u>V  $\alpha$  T/P</u>, so PV/T = constant (if moles are constant) -  $\frac{V_f P_f}{T_f} = \frac{V_i P_i}{T_i}$  and  $V_f = V_i \left(\frac{P_i}{P_f}\right) \left(\frac{T_f}{T_i}\right)$ 

Ex 5.04 Using the Combined Gas Law to Determine Volume when P and T both Change

$$-V_{f} = V_{i} \left(\frac{P_{i}}{P_{f}}\right) \left(\frac{T_{f}}{T_{i}}\right) = (82.4 \text{ cm}^{3}) \left(\frac{752 \text{ mmHg}}{760 \text{ mmHg}}\right) \left(\frac{273 \text{ K}}{293 \text{ K}}\right) = 76.0 \text{ cm}^{3} \times \left(\frac{1 \text{ mL}}{1 \text{ cm}^{3}}\right) = 76.0 \text{ mL}$$

Avogadro's Law

- Equal volumes of any gases with the same T and P contain the same number of molecules.
- The identity of the gases changes the volume only slightly due to intermolecular forces.
- Molar Volume ( $V_m$ ) is the volume (in L) of one mole of gas. That is:  $V_m$  = liters  $\div$  moles
- For an ideal gas (no intermolecular forces):  $V_m = 22.41 \text{ L/mol}$  at STP (0 °C and 1 atm)
- To create water from  $2H_{2(g)} + O_{2(g)} \rightarrow 2H_2O_{(g)}$ , 2 L of H<sub>2</sub> will react with exactly 1 L of O<sub>2</sub>. How many L of water vapor are formed?

Ideal Gas Law

- V = (constant)(T/P) if n is constant. V is proportional to moles, so V/n = (constant)(T/P).
- $V_m = V/n = (R)(T/P)$ , where the molar gas constant (R) is 0.08206  $\frac{L \cdot atm}{mol \cdot K}$ .
- The equation rearranges to PV = nRT.
- Boyle's, Charles', and Avogadro's Laws can all be derived from the ideal gas law.

Ex 5.05 Rearrange the Ideal Gas Law to determine Volume as a function of n, P, and T

$$V = \frac{nRT}{P} = \frac{0.00324 \text{ mol} \times 0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \times 294 \text{ K}}{0.952 \text{ atm}} = 0.0821 \text{ L} \times \left(\frac{1000 \text{ mL}}{1 \text{ L}}\right) = 82.1 \text{ mL}$$

Ex 5.06 Use the Ideal Gas Law to Determine the Moles and Mass for 25.4 L of  $CO_{2(g)}$  at STP

- 
$$n = \frac{PV}{RT} = \frac{(1.00 \text{ atm})(25.4 \text{ L})}{(0.08206 \frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}})(273 \text{ K})} = 1.13 \text{ mol of } \text{CO}_{2(g)}$$

- mass =  $(n)(M_m) = (1.13 \text{ mol})(44.0 \text{ g/mol}) = 49.7 \text{ g}$ 

Gas Density

- <u>Density</u> (d) in g/L is mass/volume:  $d = \frac{m}{V} = \frac{(M_m)(n)}{V} = \frac{(M_m)(P)}{RT}$ The equation rearranges to  $P(M_m) = dRT$  and  $M_m = \frac{dRT}{P}$ .

Ex 5.07 Determine the Density of  $CO_{2(g)}$  at 35  $^{o}C$  (308 K) and 1.08 atm

- 
$$d = \frac{(P)(Mm)}{RT} = \frac{(1.08 \text{ atm})(44.00 \frac{g}{mol})}{(0.08206 \frac{L \cdot atm}{mol \cdot K})(308 \text{ K})} = 1.88 \text{ g/L}$$

Ex 5.08 Determine M<sub>m</sub> for a 1.338 g sample of Xe gas in 250.0 ml at 19 °C and 742 mmHg

- V = 0.2500 L, T = 19 °C + 273 = 292 K, and P = 742 mmHg 
$$\times \frac{1 \text{ atm}}{760 \text{ mmHg}} = 0.976 \text{ atm}$$

- 
$$d = m/V = (1.338 \text{ g})/(0.2500 \text{ L}) = 5.352 \text{ g/L}$$

- 
$$M_m = \frac{dRT}{P} = \frac{\left(5.352 \frac{g}{L}\right) \left(0.08206 \frac{L \cdot atm}{mol \cdot K}\right) (292 \text{ K})}{0.976 \text{ atm}} = 131 \frac{g}{mol} = 131 \frac{amu}{atom}$$

Stoichiometry with Gas Volumes

- Determine mole ratios from coefficients in the **balanced reaction**.
- Use the ideal gas law to determine moles.

Ex 5.09 Use Stoichiometry to Determine Amounts of Reactants

- $Na_{2}CO_{3(s)} + 2HCl_{(aq)} \rightarrow 2NaCl_{(aq)} + H_{2}O_{(L)} + CO_{2(g)}$
- V = 95.2 mL of CO<sub>2</sub>, P = 743 mmHg  $\times \frac{1.00 \text{ atm}}{760 \text{ mmHg}} = 0.978 \text{ atm}, \text{ T} = 20 \text{ }^{\circ}\text{C} + 273 = 293 \text{ K}$

$$- n = \frac{PV}{RT} = \frac{(0.978 \text{ atm})(0.0952 \text{ L})}{(0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}})(293 \text{ K})} = 0.00387 \text{ mol } \text{CO}_{2(g)}$$

- $(0.00387 \text{ mol } \text{CO}_2) \left(\frac{2 \text{ mol } \text{HCl}}{1 \text{ mol } \text{CO}_2}\right) = 0.00774 \text{ mol } \text{HCl}_{(aq)}$  $(0.00387 \text{ mol } \text{CO}_2) \left(\frac{1 \text{ mol } \text{Na2CO}_3}{1 \text{ mol } \text{CO}_2}\right) = 0.00387 \text{ mol } \text{Na_2CO}_{3(s)}$
- $(0.00387 \text{ mol } Na_2CO_{3(s)})(106.0 \text{ g/mol}) = 0.410 \text{ g } Na_2CO_{3(s)})$

Gas Mixtures – Law of Partial Pressures (J. Dalton)

- Each gas in a mixture occupies the total volume and exhibits its own partial pressure.
- Each component follows the ideal gas law  $(P_{partial} = nRT/V)$ .
- Sum of partial pressures for all components equals the total pressure. -
- $P_{total} = P_a + P_b + P_c + \dots$

Mole Fraction is the moles of a component divided by the total moles in the mixture.

- Partial pressure is directly proportional to the number of moles because  $P_a = (n_a)(RT/V)$ .
- So, the mole fraction and the pressure fraction for gases are identical.
- $X_a = \frac{\text{moles of component } a}{\text{total of all moles in mixture}} = \frac{n_a}{n_{total}}$ Mole Fraction: D

- Pressure ratio is identical: 
$$X_a = \frac{\text{pressure of component a}}{\text{total pressure of mixture}} = \frac{n_a(\overline{v})}{n_{\text{total}}(\frac{RT}{v})} = \frac{P_a}{P_{\text{total}}}$$

Ex 5.10 Determine the Partial Pressure and Mole Fraction of Oxygen in Air

- A 1.000 L sample of dry air at 25.00  $^{\circ}$ C and 778 mmHg contains 0.281 g O<sub>2</sub>.
- $P_{\text{total}} = 778 \text{ mmHg} \times \frac{1 \text{ atm}}{760 \text{ mmHg}} = 1.024 \text{ atm}$  $T = 25.00 \ ^{\circ}C + 273.15 = 298.15 \ K$

- 
$$n_{O2} = 0.281 \text{ g } O_2 \times \frac{1 \text{ mol}}{32.00 \text{ g}} = 0.00879 \text{ mol } O_2$$

$$-P_{O2} = \frac{n_{O2}RT}{V} = \frac{(0.00879 \text{ mol})(0.08206 \frac{\text{L atm}}{\text{mol K}})(298.15 \text{ K})}{1.000 \text{ L}} = 0.215 \text{ atm}$$

$$X_{O2} = \frac{P_{O2}}{P_{total}} = \frac{0.213 \text{ atm } O2}{1.024 \text{ atm } total} = 0.210 \frac{\text{atm } O2}{\text{atm } total}$$

$$- X_{O2} = \frac{n_{O2}}{n_{total}} = 0.210 \frac{\text{mol } O2}{\text{mol } total} \qquad n_{total} = \frac{n_{O2}}{X_{O2}} = \frac{0.00879 \text{ mol } O2}{0.210 \text{ mol } O2 / \text{mol } total} = 0.0419 \text{ mol } total$$

Gas collected over water

- All volatile liquids, including water, have a vapor pressure.
- Vapor pressure is caused by molecules leaving the liquid phase for the gas phase.
- The vapor pressure is measured when the substance's gas is in equilibrium with its liquid.
- Vapor Pressure increases with T, but not linearly.
- Vapor Pressure of Water (Pwater vapor) versus T can be found in both Table 9.2 and Figure 9.22 of the text, as well as in Appendix C of the lab manual.
- Partial Pressure of a gas (Pgas) collected over water is Ptotal Pwater vapor because Dalton's law tells us that  $P_{total} = P_{gas} + P_{water vapor}$ .

## Ex 5.11 Determine the Mass for 98.6 mL of Carbon Dioxide Gas Collected over Water at 21 °C

- $P_{total} = 771.8 \text{ mmHg}$  and  $P_{water} = 18.7 \text{ mmHg at } 21 \text{ }^{\circ}\text{C} (294 \text{ K})$
- $P_{CO2} = P_{total} P_{water} = 771.8 18.7 = 753.1 \text{ mmHg} \times \frac{1 \text{ atm}}{760 \text{ mmHg}} = 0.991 \text{ atm}$
- $n_{\rm CO2} = \frac{P_{\rm CO2}V}{\rm RT} = \frac{(0.991 \text{ atm})(0.0986 \text{ L})}{\left(0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(294 \text{ K})} = 0.00405 \text{ mol}$
- mass =  $(n)(M_m) = (0.00405 \text{ mol})(44.0 \text{ g/mol}) = 0.178 \text{ g CO}_2$

## Kinetic – Molecular Theory of Gases

Developed by R. Hooke, D. Bernoulli, John Waterston, James Maxwell, and Ludwig Boltzmann There is a set of postulates that leads to the ideal gas law.

- 1. The size of gas molecules is negligible compared with the distance between them. Most of gas volume is empty space.
- 2. Molecules move in straight lines, in all directions, at various speeds. Pressure is same in all directions.
- 3. Attractive and repulsive forces of molecules are weak or negligible, except during collisions. Molecules move in straight lines at constant speeds until they collide.
- 4. When molecules collide, the collisions are elastic, which means Kinetic Energy is conserved.
- 5. Average Molecular Kinetic Energy (E<sub>k</sub>) is directly proportional to T, where  $E_k = (\frac{1}{2})mv^2$ .  $E_k \alpha$  T, so we also have  $(\frac{1}{2})mv^2 \alpha$  T.

Real Gases

- The ideal gas law can be adjusted using two coefficients (a and b).
- The coefficients are each specific to a particular substance and are listed for several substances in <u>Table 9.3</u>.
- The limitation on postulate 1 is that a small portion of the total volume is not empty because the molecules do take up some space. Less than the total volume is actually available:  $V_{real} = (V_{total} - nb)$ .
- The limitation on postulate 3 is that intermolecular attractions <u>decrease</u> the effective (measured) pressure.

The real gas pressure is higher than the effective pressure:  $P_{real} = P_{effective} + (a) \left(\frac{n}{v}\right)^2$ 

$$P_{real}V_{real} = [P + (a)\left(\frac{n}{V}\right)^{2}][V - nb] = nRT$$

This rearranges first to P + (a)  $\left(\frac{n}{v}\right)^2 = \frac{nRT}{(V-nb)}$ 

Rearrange once more to get the equation for pressure:  $\mathbf{P} = \frac{\mathbf{n}\mathbf{R}\mathbf{T}}{(\mathbf{V}-\mathbf{n}\mathbf{b})} - (\mathbf{a})\left(\frac{\mathbf{n}}{\mathbf{V}}\right)^2$ 

Ex 5.14 Determine Pressure using the Van der Waal's Equation

- Suppose there is 1.000 mole of  $H_2O_{(g)}$  in 22.50 L at 100.00  $^{\rm o}C.$
- Coefficients for H<sub>2</sub>O from <u>Table 9.3</u>:  $a = 5.46 \frac{L^2 \cdot atm}{mol^2}$  b = 0.0305 L/mol-  $\frac{nRT}{(V-nb)} = \frac{(1.000 \text{ mol})(0.08206 \frac{L \cdot atm}{mol \cdot K})(373.15 \text{ K})}{[22.50 \text{ L} (1.000 \text{ mol})(0.0305 \frac{L}{mol})]} = \frac{30.62 \text{ L} \cdot atm}{22.47 \text{ L}} = 1.363 \text{ atm}$

$$- (a)\left(\frac{n}{V}\right)^2 = (a)\left(\frac{n^2}{V^2}\right) = \left(\frac{5.46 \text{ L}^2 \cdot \text{atm}}{\text{mol}^2}\right) \left(\frac{1.000 \text{ mol}}{22.50 \text{ L}}\right)^2 = \left(\frac{5.46 \text{ L}^2 \cdot \text{atm}}{\text{mol}^2}\right) \left(\frac{1.000 \text{ mol}^2}{506.25 \text{ L}^2}\right) = 0.0108 \text{ atm}$$

-  $P = \frac{nRT}{V-nb}$  - (a)  $\left(\frac{n^2}{v^2}\right) = 1.363 \text{ atm} - 0.0108 \text{ atm} = 1.352 \text{ atm}$ Leatm

- Compare with the ideal gas law: 
$$P = \frac{nRT}{V} = \frac{1.000 \text{ mol} \times 0.08206 \frac{L^{-4111}}{\text{mol} \cdot \text{K}} \times 373.15 \text{ K}}{22.50 \text{ L}} = 1.361 \text{ atm}$$